What's a Part?
On Formalizing Part-Whole Relations

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Abstract. This paper deals with commonsense understanding of part-whole relations. A unifying definition for the different kinds of parts is proposed for one domain, that of rigid, man-made objects. First, the representation of objects via basic dimensions, reference systems and attributes is described. Then, different kinds of parts, based on the classification of Gerstl and Pribbenow (1996), are examined. Several problems with the seemingly uncomplicated notion of "part" are discussed. Modifying the representation of objects makes it possible to give one single definition of "a part P of an object O".

1 Introduction

The objective of this article is to investigate the following question: What are possible parts of objects of everyday use like cups, chairs or ball pens? The commonsense notion of part-whole relation(s) is an important aspect of knowledge representation, cognitive modeling as well as of the more technical area of knowledge engineering. The part-whole relation(s) are essential for different cognitive tasks, e.g., object recognition, reasoning about objects, and language understanding. The same hold for technical domains like model based configuration or diagnosis.

Part-whole relations are investigated from different perspectives.[1] Mereological approaches focus on calculi based on the "part-of" relation and their use in formal ontologies of different domains (e.g., Simons (1987), Smith (1982)). Linguistically oriented approaches examine the role of part-whole relations as semantic primitives and their relation to natural language expressions (e.g., Cruse (1979)). Psychological work concerns the role of parts in object recognition (e.g., Biederman (1987)) or in concept theory (e.g., Tversky (1990)). In cognitive science, classifications of part-whole relations are developed (e.g., Winston et al. (1987)) and in artificial intelligence, special formalisms for part knowledge are designed.

All approaches mentioned above concentrate on single topics. What is normally missing is a characterization of the notions of "part" and "whole" and the interaction between them. In order to achieve such a characterization, two questions must be answered:

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[1] A more detailed overview over the current approaches to part-whole research can be found in Gerstl and Pribbenow (1995).

1.) What is a part? Is there any unique definition that formalizes the (commonsense) notion of a part?
2.) How can parts be combined to wholes? In which way do the features of the part contribute to the features of the whole object?

I will try to answer the first question in this paper for a restricted domain, the domain of man-made, rigid objects without taken temporal change into account. In this short paper, I will not try to answer the second question, but give a short outlook on the problems of the notion of a whole at the end of the paper.

2 The representation of objects

This paper considers only man-made, three-dimensional objects that do not have movable parts and that do not change their form in any other way. This means that liquids like (non-frozen) water are thus not considered. Artifacts are easier to examine than natural objects because their general structure and function is clearer to us as a result of their being designed by humans. Nevertheless, there is a good chance that the results can be extended to natural object. The objects should be rigid to allow for a simple spatial description of the whole. Additionally, the exclusion of time makes it impossible to describe the motion of parts and leads automatically into a domain of rigid objects. The timeless point of view also allows to ignore the problem of identity of one individual that undergoes any change in its parts, e.g., a chair of which one leg has fallen off and replaced by a new leg to get a complete, fully functioning chair again. Even though most objects in the domain form a spatially connected whole, self-connectedness is not a necessary condition. The object can have removable parts like the cap of a bottle or the parts can be disconnected in space like in a deck of cards.

The objects in the domain are described by basic dimensions and attributes. Before I start with the analysis of parts, a first approach to the representation of objects will be given. The result of the analysis of parts will reveal whether this approach is sufficient or should be modified.

Basic dimensions

As stated above, the basic dimensions of our domain are three spatial dimensions and each object is associated with its "spatial extent", that is the subspace occupied by the object. It is possible to infer the volume, the form, and gestalt aspects like squareness from the spatial extent of an object. The extent of an object remains the same under spatial translation (shift from one position in space to another) or rotation. One result of these invariances is that extent does not take into account the position of an object. The extent of an object is normally connected but it could be scattered as in the above mentioned case of the single cards of a deck.

To each object, an own reference system consisting of three axes is assigned that reflects the canonical alignment of the object (see Figure 1b).[2] The object´s reference system is augmented by labels to make it possible to differentiate between the

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[2] For artifacts, the three axes of the object´s reference system almost always are straight and orthogonal. In the domain of natural objects, exceptions from straightness are, for example, streets or rivers with one curved axis and exceptions from orthogonality are, for example, crystals.

different sides of the object, e.g., its top and its bottom. If such canonical orientations do not exist at all as in the case of a perfect sphere or not for all axes as in the case of a pencil which lacks the top/bottom and the left/right orientation, the undetermined dimension axes are chosen arbitrarily. To distinguish object-determined from arbitrary axes, the two directions of the object-determined ones are labelled with the alignment they represent, e.g., top - bottom for the vertical axis, while the arbitrary ones do not get any labeling. An object´s reference system can be related to the basic dimensions if this relational information is needed for some task.

As shown in definition (Def 1), an object is described along the basic dimensions by its extent, its reference system, and the labels of its reference system.[3]

(Def 1): Basic representation of an object		
Basic dimensions:	$\mathbf{D} = D_1 \times D_2 \times D_3$	with Di and Dj orthogonal
Extent:	ext: {objects} --> $\boldsymbol{P}(\mathbf{D})$ a function that assigns to each object O its extent	
Ref. system:	rs: {objects} --> $\{(R_1, R_2, R_3)\}$	
Labels:	label: {objects) --> $\{(R_i,$ (L1, L2)) for each canonically aligned R_i of rs(O)}	
Object O:	$\mathbf{D} \supset$ ext(O)	the extent of an object, a subspace of $\mathbf{D}$
	rs (O)	the object´s reference system
	label (O)	the set of labels belonging to the object´s reference system

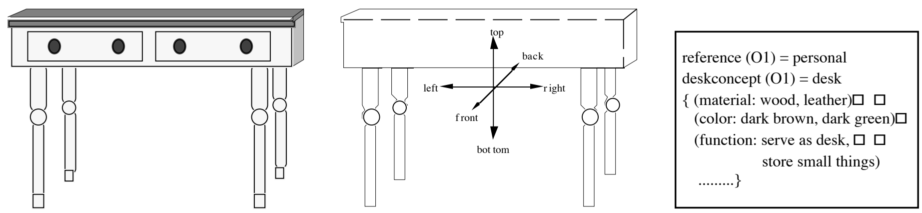
Attributes

All other knowledge about an object is given by attributes or features. As objects are concrete entities they have a unique unambiguous name or reference. They are instances of a concept from which they inherit what attributes are applicable together with the possible value range of these attributes. In the chosen domain, the most important attributes are the features "material", "color", or "function". Other possible (abstract) features are the monetary or subjective "worth", "beauty", or "dangerousness". For the characterization of an object the attribute slots inherited by the appropriate concept are filled with concrete values, e.g., the set of specific materials from which the object is built up. The basic representation of an object given in (Def 1) is now augmented by attributes as shown in (Def 2).

(Def 2): Complete representation of an object O	
Basic representation: ext(O), rs (O), label (O)	extent, reference system, and labels of O
Attributes:	
reference (O), concept (O)	unique name and concept of O
$\{(attr_i$ (O): values)}	concrete values for all attributes inherited by the concept (O)

Figure 1a shows a concrete object, my personal Biedermeier lady´s desk. Its extent and the object-determined reference system is shown in Figure 1b. My desk is an instance of the concept "desk" which is a subconcept of "table". The material is wood and leather (at the top), the colors are dark brown and dark green. Its function is to serve as a desk and to contain things in its drawers. A partial attribute description is given in Figure 1c.

[3] Similar theoretical considerations about object representation have been made by Landau and Jackendoff (1993) and van der Zee (1996).



a.) object b.) extent, reference system, labels c.) attributes

Fig. 1. Representation of my personal Biedermeier lady´s desk

3 What can be a part?

Let´s go back to the main question of the paper about the nature of the parts of an object. Examples of different kinds of parts are given which constitute different kinds of relations between a part and the corresponding whole. In this article, only examples are given. A classification of part-whole relations, its comparison with other approaches, and examples for its use can be found in the papers of Gerstl and Pribbenow (1995, 1996).[4] Further information on how the different kinds of parts can be represented and/or constructed in a mental model approach can be found in Pribbenow (1996).

In the following, for each kind of part-whole relation the underlying definition of a part is given in terms of definition (Def 2).

Structure dependent parts

1a.) the top / a leg - my desk a component of a complex object

1b.) one card - a deck an element of a collection object

The pairs under 1a.) are examples for the most typical notion of "part". The top or a leg are so-called *components* of a desk or a table where the whole is conceptualized as a complex object. Complexes are wholes with an inherent part structure, where all parts play a special role with respect to spatial, temporal, functional, and other aspects. In example 1b.), the deck is composed of a number of cards which are uniform under the chosen perspective. In this case, the whole object is a collection of undistinguishable *elements.* Components and elements are called structure dependent parts because in both cases the dismantling of the respective whole in parts is given a priori by conceptual knowledge.

Each component or element has a three-dimensional extent and an object reference system that may be different from that of the whole. It has a unique reference, is an instance of a concept, and is described by the attributes inherited from its concept. The only difference between components and elements is that components are described by different concepts while all elements of a collection by definition belong to the same class.

Some of the attributes of components and elements are the same as those that hold for the whole object, e.g., "material", "color", "function", or "worth". The values of the attributes are in general different from those of the whole, except the values of the

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4 In the examples, the quantity - mass relation (see Gerstl & Pribbenow 1995, 1996) is omitted because mass objects like liquids do not belong to the domain under consideration.

substantial attributes like "material" which are a subset of those of the whole. To account for that difference, the definition (Def 2) is modified by distinguishing between substantial and abstract attributes. An attribute is called substantial if it is defined for all –□material□– pieces of the whole object and if the attribute´s values can be determined for all materially occupied subspaces of the object´s extent. Opposite to that, abstract attributes like "function" or "beauty" are restricted in their applicabi-lity and are only defined for the whole object and – at most – certain parts of it. For example, there may exist attributes for the whole that cannot be used for all of its parts like "maximum load" for "table" but not for "table top". Therefore, the differen-ce between substantial and abstract features is not the difference between physically concrete and abstract attributes but between overall defined and restricted attributes.

(Def 2'): Modified complete representation of an object O
Attributes:

substantial ones $\{(attr_i(O): values)\}$	attribute-value pairs defined for all pieces of O
abstract ones $\{(attr_j:(O) values)\}$	attribute-value pairs defined for O or some parts of O

The rest of the definition remains the same

At first glance, components and elements can be described in nearly the same way as whole objects and definition (Def 2') seems to be a correct and exhaustive way to represent the structure dependent parts. This is true for most cases although there are exceptions like 1c.) and 1d.).

1c.) the left drawer´s interior - my desk	a component of a complex object
1d.) the keyhole - a lock	a component of a complex object

The drawer and its interior are surely important parts of my desk because they allow me to store objects like pencils and papers. The dimensions of the interior and not that of the whole drawer determine what objects can be put into the drawer. In example 1d.), the keyhole is regarded as an essential part of a lock. The two examples show that it is necessary to consider non-material components.[5] They form dependent parts of an object because they could not exist independently of the whole like a drawer or a leg. In all other ways, dependent components equal "normal" components. In order to deal with those dependent but sometimes essential parts, definition (Def□1) must be extended. The whole object must include its non-material components like the interior of rooms, drawers or cups. Therefore, the extent of an object must be defined as the sum of the materially occupied spaces and the associated spaces of an object (see Def 1'). This definition corresponds to the extent shown in Figure 1b) that included the interior of the drawers.

(Def 1'): Modified basic representation of an object

Extent:	ext: {objects} --> **D**	a function that assigns to each object O as extent its material occupied and associated spaces

The rest of the definition remains the same

Summary: Considering components and elements together with (Def 1'), a part can be defined as a subspace of the extent of the whole object but with a different reference system. A part can be described along the lines of definition (Def 2'). Each

[5] Landau and Jackendoff (1993) also consider this kind of parts. They call them "negative parts".

part has a unique reference, a concept, and is characterized by concrete values of the attributes inherited and restricted by its concept. The values of the substantial attributes are restricted by the values of the whole object.

Temporarily constructed parts

2a.) the wooden parts of my desk a portion of an object
2b.) the upper half of my desk a spatial segment of an object

In example 2a.), the wooden parts are that portion of the desk that are made out of a specific material. To compute *portions*, properties of the whole object are used to select those parts of the whole that have special values for certain attributes. To construct portions it is necessary to know the possible attributes of an object and their value distribution over the object. The upper half in example 2b.) is a *segment* of the spatial extent of the object, computed with the help of the object´s or an external (deictic) reference system and based on volume proportion.

In general, spatial segments like halves or corners are determined by the spatial properties of volume proportion and gestalt/form of whole and part while portions make use of attributes provided by the concept of the whole. Although the construction procedures are quite different, both kinds of parts can be defined as a subspace of the extent of the whole object, including non-material parts if necessary. As opposed to components and elements, segments and portions cannot be described using all features of definition (Def 2) because they are temporarily constructed by attributes or schemata and therefore are not necessarily instances of a concept.[6] In general, segments and portions can only be described by the substantial attributes of the whole object like "material" or "color", an exception is the description of a portion by the abstract attribute that was used to define it. As segments and portions are subspaces of the spatial extent it is possible to infer volume or form aspects (see section 2, "Basic dimensions").

As the considerations above show, a segment or portion can be described along the same basic dimensions as its whole and by the substantial attributes of the whole. A problem with this rather elegant and satisfying part definition arises if examples like 2c.) are considered:

2c.) the upper surface of my desk a spatial segment of an object

A surface is normally defined as a two-dimensional entity that cannot exist - like an interior - independently of the whole. A surface is a rather problematic entity: It has no volume but a form/gestalt, the attribute "material" is not applicable but "texture" and "color" normally are. If such entities are accepted as parts, the general notion of a part cannot any longer be described by the same basic dimensions as its whole because it might not be three-dimensional. There are three ways out of this dilemma:

A. Allow only three-dimensional parts and interpret a surface as a three-dimensional part with minimal thickness analog to the view of wallpaper as surface of a wall.

[6] Especially for artifacts, portions tend to correspond to one or a small number of components because most components are designed for providing one function, being made of one material, and so on.

B. Allow only three-dimensional parts and interpret a surface as an abstract construction that is not a "real" part.
C. Change the definition of a part radically to the following: A part is a zero- to three-dimensional entity embedded in the extent of the whole. If it is more than zero-dimensional it has an appropriate reference system and the substantial attributes appropriate to the given dimensions.

All alternatives have advantages and disadvantages. Alternative A implies a satisfying definition of "part" and follows the philosophy that our world has a three-dimensional nature but it does not take into account the rather abstract mathematical notion of a surface.[7] Alternative B also implies a satisfying definition but at the cost that it restricts the possible parts. Alternative C gives a liberal definition of part by which, in consequence, a part can no longer be described as a subspace of the extent of its whole. The alternative to be chosen partially depends on the application. For tasks involving mathematical or formal thinking, the abstract notion in C might be appropriate. For purely commonsense tasks, the three-dimensional nature is important and therefore C is excluded. Alternative B is not a good alternative if the whole range of part phenomena should be considered. Therefore, alternative A is chosen for the definition given in this paper.

Summary: Considering segments and portions, a part can be defined as a subspace of the extent of the whole object. Normally it does not belong to a concept and has no reference system. In general, a part can be described by the substantial attributes of the whole object where the part´s values of these attributes are a subset of the values provided by the whole.

Arbitrary parts

3.) a splinter of the top of my desk an arbitrary piece of an object

Arbitrary parts could be regarded as segments without a definite spatial characterization or as portions without a characterization via the values of some attributes. Therefore, these parts could be defined with the definition of segments and portions given above.

Given the considerations about structure dependent parts (components and elements), temporarily constructed parts (segments and portions), and arbitrary parts, the notion of a part can be determined as given in (Def 3) that is based on the definitions (Def 1') and (Def 2').

(Def 3): Definition of a part P of an object O
P is a part of O iff
(1) P is described by the basic dimensions **D** with the additional condition
$ext(O) \supset ext(P)$
(2) P is described with the same substantial attributes as the whole with the condition
$values_O \supset values_P$ for all substantial attributes

Following this definition, everything is a part that shares extent and certain attribute values. The definition does not say anything about which kind of part, e.g., a

[7] Although alternative A interprets a surface as three-dimensional, this does not prevent the surface from being conceptualized as a two-dimensional entity for some tasks.

component or an arbitrary part, a specific part belongs to. This simple definition is only possible because the extent of a whole was extented to associated, not materially occupied spaces and the possible attributes were distinguished in substantial and abstract ones.

4 Outlook

As stated in the introduction, the characterization of the commonsense notion of part-whole relations consists of two subquestions. In this paper, I have tackled the first question about the definition of the notion of "part". The second question to be discussed concerns the notion of "whole" and how a whole is constructed out of its parts. The answer to that question is a current research topic, of which only a brief outlook can be given at the moment. Following an old proverb, a whole is more than the sum of its parts. Therefore, the commmonsense notion of a (complex) whole assumed an additional structure between the different parts in order to build up the whole. This structure involves at least spatial and functional aspects of the part-structure. As a consequence, the notion and construction of a whole cannot be regarded as the mere inverse of the notion of a part.

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